

A Bayes Factor for Replications of ANOVA Results

Christopher Harms^{1,*}

1 Rheinische Friedrich-Wilhelms-Universität Bonn, Germany

* christopher.harms@uni-bonn.de

Abstract

With an increasing number of replications in psychological science, the question of how to evaluate the outcome of a replication attempt deserves careful consideration. Different approaches have been described before. Bayesian approaches allow to incorporate prior information into the analysis of the replication attempt by their design. The Replication Bayes Factor, introduced by Verhagen and Wagenmakers (2014), is outlined and extended to F -tests in multi-group, fixed-effect ANOVA designs. Derivation of likelihood functions and computational strategies are explained. Simulations and examples are presented to facilitate the understanding and in order to demonstrate the usefulness of this approach. The Replication Bayes Factor for F -tests is discussed in the context of evaluating replication attempts and compared to other approaches.

1 Introduction

The so-called "replication crisis" has led to an increased focus on matters of replicability and replication studies (Asendorpf et al., 2013). One of the central questions of a replication attempt is the final evaluation, that is, the question "Did it replicate?".

While the question by itself and the reasoning might seem easy, there is an ongoing discussion of how to actually answer it (Anderson and Maxwell, 2016; Simonsohn, 2015; Verhagen and Wagenmakers, 2014). The traditional and most commonly used approach relies merely on the evaluation of p -values and significance across manipulations, that is, the investigator checks if his replication yields the same pattern of significant results as the original study. A single focus on this approach, however, has two major limitations: First, it relies on p -values, that can easily be "hacked" in both the original and the replication study (Simmons et al., 2011). And second, it is uninformative about the true size of the effect.

Different approaches to evaluating replication results have been introduced in recent years: Simonsohn (2015) proposed an approach taking the effect size of the original study and a "minimal detectable effect" into consideration. Confidence intervals for effect size measures have been repeatedly advocated by Cumming (2012). Mixing those strategies have been recommended by Brandt et al. (2014) and Anderson and Maxwell (2016) among others. A different approach is to take a Bayesian perspective and use Bayes Factors for hypothesis testing (see Morey et al. (2016) for an introduction on Bayesian hypothesis testing). One approach from this line of thinking has been introduced by Verhagen and Wagenmakers (2014) and termed *Replication Bayes Factor*.

The present article focuses on this Bayesian perspective and extends the Replication Bayes Factor to the case F -tests in fixed-effect ANOVA designs. In order to do so, first the Bayesian perspective is outlined and how it is applied to replication studies. In the second section it is shown how the Replication Bayes Factor can be used in the context of fixed-effects ANOVAs. Simulations and example studies are followed by a discussion of the method.

2 A Bayesian Perspective of Replications

Already before the so-called "replication crisis", Bayesian approaches to data analysis have been recommended by proponents to overcome problems with traditional Null Hypothesis Significance Testing (NHST; Lindley, 1993; Edwards et al., 1963). With advances in computational capabilities and statistical software packages, Bayesian statistics today are more accessible than ever before. Since traditional NHST is still the only approach taught at many universities, however, many junior and senior researchers are yet unfamiliar with Bayesian methods. The general principles of the Bayesian framework in the context of the Replication Bayes Factor is outlined in the following section. Far more detailed and educational literature on Bayesian statistics in general and Bayes Factors in particular is available (McElreath, 2016; Rouder et al., 2012; Morey et al., 2016; Kruschke, 2015).

Fundamental and name-giving to the Bayesian approach is Bayes' theorem, which shows how conditional and unconditional probabilities are related to each other:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \quad (1)$$

There are different philosophical and practical interpretations of Bayes' theorem and Bayesian statistics. In this paper, the focus is on a process termed *Bayesian updating*: If Equation 1 is rephrased as

$$\text{Posterior} \propto \text{Likelihood} \times \text{Prior}$$

empirical data, expressed through their respective *likelihood*, can be used to update some *prior belief* to a *posterior belief*. This implicates some understanding of probabilities as "degrees of belief" or "subjective probability". It is to note, however, that many statistical applications of this have both frequentist and Bayesian interpretations. The current paper will not go into further detail about the consequences of this terminology and the philosophical implications.

Imagine a scientist, Sam, who is new to a particular field of research. She might start to read books, papers and blog articles to get an understanding of the current state of the field and learn about some popular effect. Before reading the first paper on this particular phenomenon, she might be seen as "uninformed", i.e. she believes that any outcome of an experiment is equally likely. After reading the paper, though, her beliefs probably have shifted: She might now be more inclined to believe that the effect described truly exists. However, she already knows that no single experiment is sufficient to ultimately rule out false positives, so there is still some subjective belief (larger than null) in the non-existence of the effect. With her current belief in the effect she reads another paper and, again, updates her belief about the effect according to the published results.

This example illustrates the basic idea of Bayesian updating. It also transfers well to the context of replications: as researchers we have some prior knowledge of an effect, formed through our own experiments and published research. This prior knowledge plays an important role when we consider a new study. With Bayes Factors a statistical tool is available, that tells researcher to what quantitative extent a prior belief should be updated.

The central idea is, to put in other words, to incorporate existing, previous knowledge about the world in the interpretation of new incoming evidence. This is particularly useful in the context of replications as information from an original experiment is taken into account when evaluating the results from a replication study.

Bayes Factors compare the support in some data in favor of a model 1 when compared to another model 2. They are one way to evaluate hypothesized models in the Bayesian framework in a quantitative manner. Morey et al. (2016) have described them as "quantification of statistical evidence". At its core, they are an odds ratio that is formally written as

$$BF_{10} = \frac{p(Data|H_1)}{p(Data|H_0)} \quad (2)$$

The Bayes Factor BF_{10} can be interpreted directly from its quantitative value: Values > 1 tell how much more likely the data are under the model of H_1 , while values < 1 express how much more likely the data are under the model implied by H_0 . This is in contrast to traditional p -values which cannot be interpreted in a similar continuous manner (Dienes, 2016, 2011). Bayesian hypothesis testing using Bayes Factors have become quite popular in the recent years as an addition or alternative to the traditional Null Hypothesis Significance Testing using p -values (e.g. Rouder et al., 2012; Rouder and Morey, 2012; Rouder et al., 2009).

2.1 Bayes Factors for Replications

The *Replication Bayes Factor* introduced by Verhagen and Wagenmakers (2014) is a way to use the Bayesian framework in the context of replications and quantify the result of a replication attempt. It defines two models that are characterized by two positions and their respective prior beliefs about some effect size measure δ :

1. The first model, H_0 , is the position of a *skeptic*, who does not believe in the original finding. Formally, $H_0 : \delta = 0$.
2. The second position is one of a *proponent* of the effect. The proponent has some belief in the original study and the postulated effect: $H_r : \delta \approx \delta_{orig}$. That means, the posterior distribution of the original study is used as a prior distribution to the replication attempt.

The Replication Bayes Factor B_{r0} can then be formally described as (Verhagen and Wagenmakers, 2014, p. 1461):

$$B_{r0} = \frac{p(Y_{rep}|H_r)}{p(Y_{rep}|H_0)} \quad (3)$$

Where Y_{orig} denotes the data from the original study and Y_{rep} the data from the replication attempt.

But, what exactly is "the data"? Ideally, likelihoods would be calculated based on a specific probabilistic model using the raw data from both studies. However, often only summary and test statistics are available for the original study. Thus, in the outlined approach it is desired to use only reported test statistic of the original (Y_{orig}) and the replication study (Y_{rep}) respectively. Verhagen and Wagenmakers (2014) have described the procedure for calculating B_{r0} for the case of t -tests in both the original study and the replication attempt. Their intuition is outlined next as it is the basis for the extension to F -tests.

First, the likelihoods for the two contrasted perspectives are to be defined and calculated: For the skeptic (the nominator term in the Bayes Factor), the likelihood is represented by the central t -distribution evaluated at the observed t -value: $p(Y_{rep}|H_0) = t_{df}(t_{obs})$. This makes the skeptic's position a point hypothesis.

For the proponent, the information from the original study is used. That means, from a Bayesian perspective, the posterior distribution of the original is to be defined first. Starting with a uniform prior distribution for the original study, the posterior of a study with a t -test is a mixture of noncentral t -distributions. This distribution was described as Λ' -distribution by Lecoutre (1999) and can be approximated through a normal distribution.¹ This posterior distribution is then used as a prior to the replication study, denoted by $p(\delta|H_r)$. For the denominator of the Bayes Factor, the computation of a marginal likelihood across all possible values of δ is required:

$$p(Y_{rep}|H_r) = \int p(Y_{rep}|\delta, H_r)p(\delta|H_r)d\delta \quad (4)$$

To approximate the marginal likelihood, Verhagen and Wagenmakers (2014) repeatedly drew samples from the prior and calculated the average of the term.

This completes the necessary steps to calculate the Bayes factor and leaves us with this formular for the Replication Bayes Factor in the case of t -tests:

$$B_{r0} = \frac{\int p(Y_{rep}|\delta, H_r)p(\delta|H_r)d\delta}{t_{df}(t_{obs})} \quad (5)$$

In their paper, Verhagen and Wagenmakers (2014) use simulation studies and examples to show the usefulness of the Replication Bayes Factor and to compare it to other Bayes Factors that might also be used for replication studies. Since many studies in psychological research are not one- or two-sample designs but ANOVA designs, there seems to be a need to extend this approach to other tests such as F -tests. The following section show how to define and calculate the Replication Bayes Factor for F -tests in fixed-effects ANOVA designs.

3 Replication Bayes Factor for F -tests

The following section extends the Replication Bayes Factor to F -tests in the context of ANOVA designs and deduces the steps necessary to calculate it.

¹The technical details of two approximation approaches are explained in the appendix of Verhagen and Wagenmakers (2014).

The general idea and the contrasted perspectives of the Replication Bayes Factor should remain unchanged. So, again, two hypotheses are defined:

- The position of a *proponent*, H_r , with a belief of $f^2 \approx f_{orig}^2$.
- The position of a *skeptic*, H_0 , who expects $f^2 = 0$.

As effect size measure for the case of F -tests Cohen's f^2 was selected since it can be calculated from a given F -value and has some conveniently simple relationship to the noncentrality parameter λ of the noncentral F -distribution (Steiger, 2004) in the case of fixed-effects models. Steiger (2004) and Lakens (2013) have detailed various effect size measures and compared their usefulness in different settings.

The skeptic's position as a point hypothesis is again simply given by the central F -distribution evaluated at the observed F -value in the original study: $p(Y_{rep}|H_0) = F_{df_{effect}, df_{error}}(F_{rep})$.

While the deduction of the proponent's position is given in a similar manner as in the case of t -tests, some differences exist. First, the posterior distribution of the original study with available $F_{orig}(df_1, df_2)$ needs to be defined, so it can then be used as a prior to the replication study.

3.1 Finding the Posterior from the Original Study

Again, an flat, uniform prior on the effect size f^2 is used for the original study, i.e. all positive values are equally likely before the first study is conducted. Through Bayes' rule it is known:

$$\text{Posterior} \propto \text{Likelihood} \times \text{Prior}$$

Since the prior is uniform, the posterior distribution is equal to the normalized likelihood, which in turn is a mixture of noncentral F -distributions.

To find the noncentrality parameter for each of the distributions in the mixture, the observed F -value F_{obs} will be used: Entering $\eta_p^2 = \frac{F_{obs} \times df_{effect}}{F_{obs} \times df_{effect} + df_{error}}$ (Lakens, 2013) in $f^2 = \frac{\eta_p^2}{1 - \eta_p^2}$ (Steiger, 2004) gives us a way to calculate f^2 directly from observed value F_{obs} and known df_{effect} and df_{error} :

$$f^2 = \frac{F_{obs} \times df_{effect}}{df_{error}} \quad (6)$$

From a given effect size f^2 and known total sample size N the noncentrality parameter λ of the noncentral F -distribution follows through

$$\lambda = f^2 \times N \quad (7)$$

The (non-normalized) likelihood \mathcal{L} is then given by:

$$\mathcal{L}(\lambda|Y_{orig}) = F_{\lambda}(df_{effect}, df_{error}) \quad (8)$$

$$\Rightarrow \mathcal{L}(f^2|Y_{orig}) = F_{f^2 \times N}(df_{effect}, df_{error}) \quad (9)$$

Leading to the posterior for f^2

$$p(f^2|Y_{orig}) = \frac{\mathcal{L}(f^2|Y_{orig})p(f^2)}{\int \mathcal{L}(f^2|Y_{orig})p(f^2) df^2} \quad (10)$$

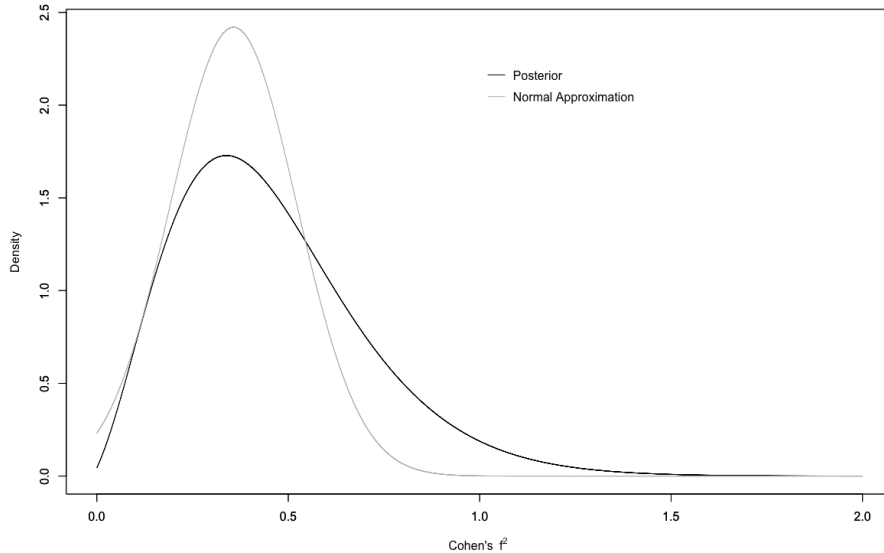


Figure 1: Normal approximation (grey line) for the true posterior distribution (black line) of Cohen's f^2 . $F_{obs}(1, 28) = 10.0$ in a one-way ANOVA design with 2 groups and 30 participants.

As the prior $p(\lambda) = p(f^2)$ is uniform and independent of values of λ or f^2 , this is equal simply to the normalized likelihood function.

While in the case of a mixture of noncentral t -distributions, the resulting posterior distribution can be approximated through a normal distribution, this does not yield a good approximation in our case. Visually, this can be seen from the example in Figure 1: the resulting posterior distribution is positively skewed and the tails do not match the normal distribution. The posterior distribution for the case of a F -test was named Λ^2 -distribution by Lecoutre (1999) and could be calculated through an iterative algorithm introduced by Poitevineau and Lecoutre (2010).

However, there is also another way to approximate a distribution and generate random independent and identically distributed samples from it (what will later be done to approximate the marginal likelihood in the Bayes Factor): The Metropolis-Hastings algorithm, a "Markov Chain Monte Carlo" (MCMC) technique, allows to sample directly from the posterior distribution in cases like this where the distribution might be inconvenient to handle otherwise (Chib and Greenberg, 1995). The Metropolis-Hastings algorithm takes a random walk through the parameter space, i.e. all possible values for λ , using a proposal distribution for the next state at which the procedure is then repeated. The introduction of MCMC techniques such as Metropolis-Hastings and other algorithms have led to new possibilities in the application of Bayesian statistics Hitchcock (2003); Han and Carlin (2001). In this context, statistical packages such as JAGS or Stan (Gelman et al., 2015) are commonly used. The present case, however, is rather simple in its nature as it only involves a single parameter and an uni-

modal posterior distribution. Thus, the Metropolis-Hastings algorithm should be sufficiently efficient.²

One of the benefits of Metropolis-Hastings is, that it directly gives samples from the posterior distribution, which can directly be used to approximate the marginal likelihood for the proponent’s model, H_r , by averaging the likelihood over the prior samples. This method of approximating the marginal likelihood was used by Verhagen and Wagenmakers (2014) and was reiterated above. It should be noted, however, that this is not the best or most efficient way to calculate the marginal likelihood. In a comparative review Bos (2002) showed that this approach can be unstable in some cases and is generally inefficient. Chib and Jeliazkov (2001) detailed a method to use the output from Metropolis-Hastings to approximate the marginal likelihood in a more efficient way when parameters can be separated into blocks. But, again, as the present case involves only one parameter and is generally simple in nature, the approximation using samples from the prior distribution (i.e. the posterior of the original study) and averaging the likelihood times the prior across all samples from the prior is considered sufficiently precise for the question at hand.

Next, the steps to calculate the Replication Bayes Factor for F -tests in fixed-effect ANOVAs are summarized.

3.2 Calculating the Replication Bayes Factor for F -tests

The Replication Bayes Factor was defined as (Verhagen and Wagenmakers, 2014):

$$B_{r0} = \frac{p(Y_{\text{rep}}|H_r)}{p(Y_{\text{rep}}|H_0)} \quad (11)$$

$$= \frac{\int p(Y_{\text{rep}}|\delta, H_r)p(\delta|Y_{\text{orig}}) d\delta}{p(Y_{\text{rep}}|H_0)} \quad (12)$$

In the case of ANOVA designs this transforms into

$$B_{r0} = \frac{\int F_{df_{\text{effect}}, df_{\text{error}}, f^2 \times N}(F_{\text{rep}})p(f^2|Y_{\text{orig}}) df^2}{F_{df_{\text{effect}}, df_{\text{error}}}(F_{\text{rep}})} \quad (13)$$

By drawing M random samples from the posterior $p(f^2|Y_{\text{orig}})$ the marginal likelihood and the Bayes Factor can be approximated:

$$B_{r0} \approx \frac{1}{M} \sum_i^M \frac{F_{df_{\text{effect}}, df_{\text{error}}, \lambda^{(i)}}(F_{\text{rep}})}{F_{df_{\text{effect}}, df_{\text{error}}}(F_{\text{rep}})}, \lambda^{(i)} \sim \frac{p(f^2|Y_{\text{orig}})}{N_{\text{rep}}} \quad (14)$$

The resulting Bayes Factor can then be interpreted based on its quantitative value: $B_{r0} > 1$ is evidence in favor of the proponent’s hypothesis, i.e. evidence in favor of a true effect of a similar size, while a $B_{r0} < 1$ is evidence against a true effect of similar size. The more the Bayes Factor deviates from 1, the stronger the evidence. It might be helpful to use the commonly used boundaries of 3 and

²In the supplemental R script, the `MCMCmetrop1R` function from the `MCMCpack` package was used.

$\frac{1}{3}$ for sufficient evidence: $\frac{1}{3} < B_{r0} < 3$ is weak evidence for either hypothesis (Jeffreys, 1961, p. 432) and should lead the researcher to collect further data to strengthen the evidence.³

4 Simulation Studies

The following section builds on the Replication Bayes Factor for F -tests outlined in the previous sections and aims to answer two questions using simulation studies: First it is shown, that approximation to the original's posterior distribution using the Metropolis-Hastings algorithm yields similar results as the normal approximation by Verhagen and Wagenmakers (2014) for the case of t -tests. Second, the relationship between the Replication Bayes Factors for t - and F -tests is investigated.

4.1 Simulation 1

Verhagen and Wagenmakers (2014) have shown that a normal approximation is very close to the true posterior distribution of an original study with a given t -value. However, for the case of F -tests this is not true as shown above. Thus, the Metropolis-Hastings algorithm (MH) was introduced to generate samples from an approximation of the posterior distribution. Using the resulting samples from the MH algorithm as prior samples allows to approximate the marginal likelihood in the Replication Bayes Factor.

To show that this in fact leads to a correct value of the Replication Bayes Factor, simulations were conducted. In each simulation two independent samples are randomly generated each for an original and a replication studies. The replication attempt is then quantified using (a) the Replication Bayes Factor for t -tests by Verhagen and Wagenmakers (2014) using a normal approximation and (b) the Replication Bayes Factor for t -tests using the M-H algorithm. If the resulting Bayes Factors are equal or at least very similar to each other (some random variation is to be expected from random sampling), this is some technical proof of concept to the introduced approach.

The simulations were set up as follows: For four different sample sizes in an original study ($n_{orig} = 10, 15, 20$ and 50 per group), three different group sizes in a replication study ($n_{rep} = 20, 50, 100$ participants per group) and six different true population effect sizes ($d = 0, 0.1, 0.3, 0.5, 0.7$ and 1) 100 runs each are simulated, if $n_{rep} \geq n_{orig}$. For each simulated pair of original and replication, samples from a normal distribution with means $\mu_0 = 0$ and $\mu_1 = d$ and standard deviation $\sigma = 1$ for both the original and the replication study are generated. For both studies then, t -values and two Replication Bayes Factors are calculated: First, the Replication Bayes Factor for two-sample t -tests according to Verhagen and Wagenmakers (2014) using a Normal approximation to the posterior distribution of the original study and second, the same Bayes Factor using Metropolis-Hastings.

The results of the simulations are shown in Figure 2: each shape represents two Bayes Factors from a single simulation run. The red lines indicate the

³It is a notable feature of Bayes Factors to allow for sequential testing as more data is collected (Schönbrodt et al., 2015; Edwards et al., 1963). Another property of the Bayes Factor that is not valid for p -values.

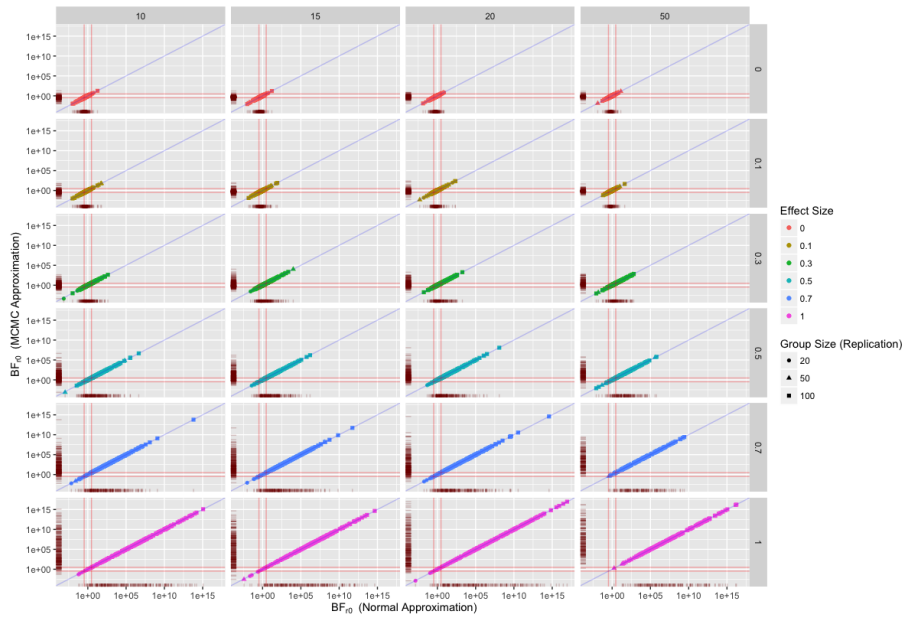


Figure 2: Results for simulation study 1: 100 simulated runs of one original and one replication study with two independent groups in different scenarios, evaluated with B_{r0} using the normal versus MCMC approximation to the posterior of the original study. Each symbol represents a single simulation run, i.e. the Bayes Factors for one original and one replication study. Red lines indicate $B_{r0} = 3$ and $B_{r0} = 1/3$, blue line indicates equality of both approaches. Axes are \log_{10} -scaled. Results for both methods are very similar ($r = .99982$, across all scenarios).

common boundaries of 3 and 1/3 for interpretation of the Bayes factors. The blue line represents the exact equality of the two Bayes Factors. As can be seen from visual inspection and numerically from $r = .99982$ (across all 6600 runs), both methods correlate nearly perfectly. While the mean difference between the two approaches seems to be very high ($-1.20173 \cdot 10^{12}$), this is mainly driven by the large Bayes Factors for configurations with $d = 1$. More informative is the mean ratio, which is close to 1 ($\frac{B_{r0, norm}}{B_{r0, MCMC}} = 1.00692$). This indicates, that the Metropolis-Hastings algorithm leads to very similar results as the original Replication Bayes Factor using a normal approximation to the original study's posterior distribution. In cases where a normal approximation is not available – as in the case for a Replication Bayes Factor for F -tests from a fixed-effect ANOVA –, Markov Chain-Monte Carlo methods such as Metropolis-Hastings thus are a useful and valid alternative.

4.2 Simulation 2

In the context of significance testing, it is a known relationship that the F -tests from a one-way ANOVA with two groups yield the same p -values as a two-sample t -test, when $F = t^2$ is used. Accordingly, the Replication Bayes Factor for F -tests should have a similar magnitude as the Replication Bayes Factor for t -tests when used for the same dataset.

However, from investigation of the Likelihood functions it is apparent, that exactly identical results are not to be expected. This is (a) because δ and f^2 are different types of effect size measures and (b) because the Λ' - and Λ^2 -distributions are different in shape (see Figure 3). Further, as samples are randomly sampled from posterior-becoming-prior distribution, some random variance in the resulting B_{r0} is again to be expected. It is, nevertheless, expected that the magnitude in both cases is very similar, so the same conclusions would be reached.

The simulated samples and t -values from simulation 1 are re-used. That is, there are 100 simulations for four different sample sizes in the original ($n_{orig} = 10, 15, 20$ and 50 participants per group) and three sample sizes in the replication study ($n_{rep} = 20, 50, 100$, whereas the simulation is only performed if $n_{rep} \geq n_{orig}$) and six different effect sizes ($d = 0, 0.1, 0.3, 0.5, 0.7$ and 1). This time, the Replication Bayes Factor for two-sample t -tests with the Replication Bayes Factor for F -tests outlined in this article are compared. In particular, each $F_{observed} = t_{observed}^2$.

As can be seen from Figure 4, again, both Bayes Factors are strongly correlated ($r = .99270$ across all 6600 runs) as expected. What is barely visible in the figure, however, is that the ratio between the two Bayes Factors is close to 2 ($\frac{B_{r0,t}}{B_{r0,F}} = 2.10952$). That means, that the Replication Bayes Factor is about half the size when using the F -statistic instead of the t -statistic.

This is a direct consequence of the transformation performed and to be expected: The F -statistic does not convey information about the direction of the difference anymore, something the t -statistic does through its sign. It is easy to see that the Bayes Factor therefore cannot represent the same strength of evidence. The consequences are further investigated in the Discussion.

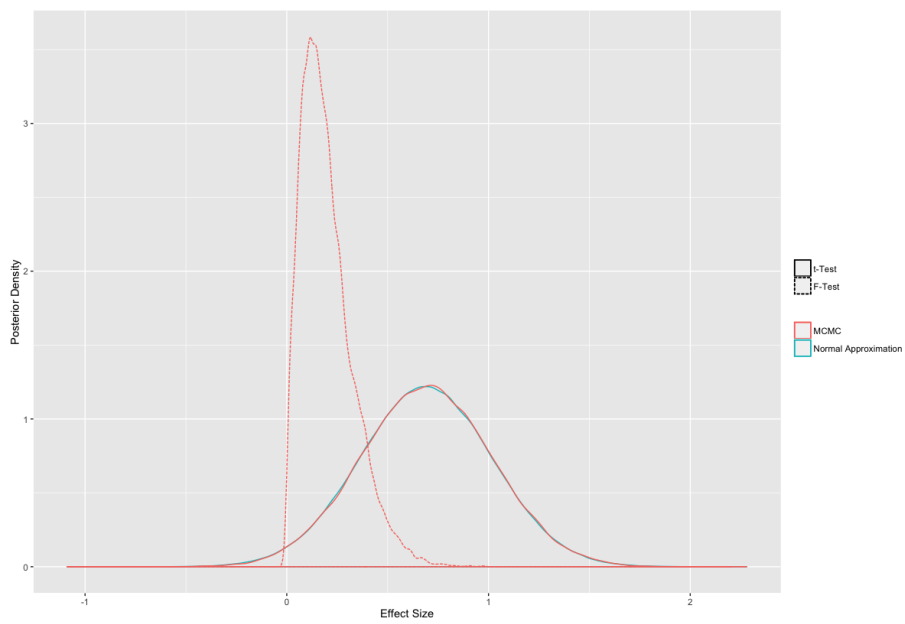


Figure 3: Comparison of posterior distributions: The posterior of an original, two-sample study with two groups, $n = 20$ each, resulting in $t_{obs} = 2.18$ is plotted with solid lines. Normal approximation (blue) and Metropolis-Hastings (red) come to identical estimates of the true posterior. Plotted with a dashed line is the posterior distribution of the original study when $F_{obs} = t_{obs}^2 = 4.7524$ is used.

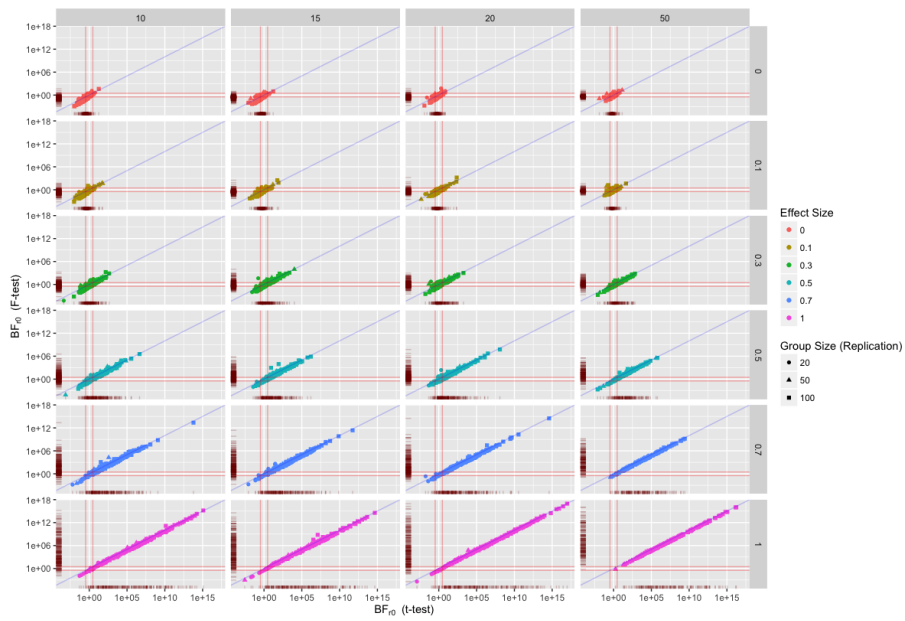


Figure 4: Results for simulation study 2: 100 simulated runs of one original and one replication study with two independent groups in different scenarios, evaluated with B_{r_0} for t - versus F -tests both using Metropolis-Hastings to sample from the posterior of the original study. Each symbol represents a single simulation run, i.e. the Bayes Factors for one original and one replication study. Red lines indicate $B_{r_0} = 3$ and $B_{r_0} = 1/3$, blue line indicates equality of both approaches. Axes are \log_{10} -scaled. Results for both methods are very similar ($r = .99270$, across all scenarios).

5 Examples

In this section the Replication Bayes Factor for F -tests is applied to two example replication attempts.

5.1 Example 1

The first example is the original study by Goschke and Dreisbach (2008) on "conflict-triggered goal shielding". Their study was replicated as part of the "Reproducibility Project: Psychology" (Open Science Collaboration, 2015) by Marco Perugini and Giulio Constantini from the University of Milan-Bicocca. The data, materials and replication report are available at <https://osf.io/pnius/>.

Goschke and Dreisbach (2008) found a significant two-way interaction between the independent variables *Compatibility* (compatible vs. incompatible) and *Prospective Memory (PM) cue dimension* on the "mean proportion of missed PM cues" in a sample of 40 subjects ($F(1, 38) = 6.21$, $p_{rep} = .927$ which equals $p = .0172$, $\eta_p^2 = .140$ which corresponds to $f^2 = .163$). The replication by Perugini & Constantini did also find a significant interaction between *Compatibility* and *PM cue dimension* on the dependent variable ($F(1, 93) = 18.94$, $p = 3.46 \cdot 10^{-5} < .001$, $\eta_p^2 = 0.169$ which corresponds to $f^2 = .203$) in their sample of 95 participants. This was considered a successful replication in the context of the Reproducibility Project.⁴

Using the reported teststatistics, the Replication Bayes Factor outlined in this article can be calculated. This yields $B_{r0} = 1654.096$, which is overwhelming evidence in favor of the proponent's position and evidence for the existence of a true effect. In contrast to simply comparing p -values, the success of the replication can be expressed in quantitative terms. Assuming that the experiment in fact provoked the same underlying effect in both studies and data were collected as outlined in the replication report, researchers should (ideally) now have a stronger belief in the reported effect and its size.

5.2 Example 2

For the second example another replication from the "Reproducibility Project" was used. The original study was conducted by Williams and Bargh (2008) and investigated cues of "spatial distance on affect and evaluation". The replication was performed by Jennifer Alana Joy-Gaba (Virginia Commonwealth University), Russ Clay (University of Richmond) and Hayley Cleary (Virginia Commonwealth University). The replication data, materials and final report are available at <https://osf.io/vnsqg/>.

In study 4 of the original paper, Williams and Bargh (2008) have primed 84 participants in three different conditions. They hypothesized that different primes for spatial distance will effect evaluations of perceived "closeness" to siblings, parents and hometown. The dependent variable was an index of ratings to those three evaluations. They found a significant main effect of priming on the "index of emotional attachment to one's nuclear family and hometown"

⁴The replication found interactions not present in the original study and was not an exact replication of the original study. However, the method was similar enough to compare the test statistics and use the Replication Bayes Factor. See Discussion for details.

($F(2, 81) = 4.97$, $p_{rep} = .95$ which equals $p = .009$, $\eta_p^2 = .11$ which corresponds to $f^2 = .124$).

The replication by Joy-Gaba, Clay and Cleary did not find the same main effect in a sample of 125 participants ($F(2, 122) = .24$, $p = .79$, $\eta_p^2 = .003919$ or $f^2 = .00393$). Based on the p -values they concluded, that the replication was not successful. In fact, if there was truly an effect $\eta_p^2 > 0$ and the replication was a better (unaffected by publication bias) estimate of the true effect size, neither the original nor the replication study would have had a chance to detect the effect as they were helplessly underpowered.

But how much more likely is it, that the effect does in fact not exist? This is the answer the Replication Bayes Factor can give: $B_{r0} = 0.031$. This means, the data is about 32 times more likely under the hypothesis that the true effect size is 0.

6 Discussion

The Replication Bayes Factor for F -tests in fixed-effect ANOVA designs developed in this paper is an extension of the work by Verhagen and Wagenmakers (2014). It utilizes a Bayesian perspective on replications, namely using the results and uncertainty from the original study in the analysis of a replication attempt. The approach outlined in this paper adapts the Replication Bayes Factor from t - to F -tests and can be used to further extend the BF_{rep} to other tests as well and also to cases where different tests are to be compared.

The simulation studies have shown, that (a) the Metropolis-Hastings algorithm is an adequate way to sample from the posterior distribution of the original study and (b) the Replication Bayes Factor for F -tests is generally similar in magnitude to the Replication Bayes Factor for t -tests when comparing two independent groups/samples. The two examples finally should have facilitated the understanding of the Replication Bayes Factor for F -tests and shown that it is simple to apply it to a given replication attempt. R scripts for the computation of the Replication Bayes Factors and for reproduction of the simulations and examples from this paper are available on the first author's website: . . .⁵

As outlined in the explanation of the Replication Bayes Factor for F -tests above, Cohen's f^2 was chosen as effect size measure. This limits its application to fixed-effects ANOVAs with approximately equal cell sizes since the noncentrality parameter λ of the noncentral F -distribution is calculated differently in cases of unbalanced designs or repeated measures ANOVAs. Steiger (2004) outlines more complicated cases of ANOVA designs such as random-effects models and Repeated Measures ANOVAs. Further information is then required to determine an effect size based on the observed F -statistic (e.g. cell sizes and specific effects α_j).

In the context of fixed-effect ANOVA designs, different F -statistics can generally be calculated. Of course, the Replication Bayes Factor can only take into account the information contained in the statistics used for its computation. The omnibus F -test for example does not contain information about either the direction or the location of a relevant difference in means. This is the reason, the ration between the Replication Bayes Factors for t - and F -tests was about 2 in the second simulation study.

⁵URL will be inserted after the anonymous peer-review.

It should further be obvious, that the Replication Bayes Factor as detailed and used here can only be applied in cases where the replication study is sufficiently close to the original study from a theoretical and from an analytical point of view. If, for example, different aspects of the effect are investigated in the original and the replication study, perhaps using different manipulations, it might not be sensible to use the Replication Bayes Factor. This is not a design problem of the Bayes Factor but a general problem that also effects the simple counting of significant results or a perspective using confidence intervals. In each case researchers have to make sensible assumptions on the generalizability of an empirical finding. This is a facet of the replicability/reproducibility debate that has not yet been covered in depth.

The Replication Bayes Factor introduced by Verhagen and Wagenmakers (2014) and extended herein is one further index to evaluate the results of a replication attempt. It is, obviously, not able to cover all questions and pitfalls in the analysis of a replication. Instead it is a way to formally and transparently integrate prior findings in the analysis within the Bayesian framework and allows to quantitatively assess the gained evidential value. It is further easy to apply to frequentist results as it uses reported test statistics from the original and replication study. To cover replications comprehensively, however, researchers have to use different tools depending on the question asked. No single statistical index is sufficient to globally assess the quality of a study or a theory, this is true not only for p -values (Wasserstein and Lazar, 2016) but also for Bayes Factors.

Acknowledgements

I thank Dr. André Beauducel for his remarks on an early version of this manuscript, that helped to improve its structure and content.

Conflicts of Interest

None.

References

- Anderson, S. F. and Maxwell, S. E. (2016). There’s more than one way to conduct a replication study: Beyond statistical significance. *Psychological Methods*, 21(1):1–12.
- Asendorpf, J. B., Conner, M., De Fruyt, F., De Houwer, J., Denissen, J. J. A., Fiedler, K., Fiedler, S., Funder, D. C., Kliegl, R., Nosek, B. A., Perugini, M., Roberts, B. W., Schmitt, M., van Aken, M. A. G., Weber, H., and Wicherts, J. M. (2013). Recommendations for Increasing Replicability in Psychology. *European Journal of Personality*, 27(2):108–119.
- Bos, C. S. (2002). A Comparison of Marginal Likelihood Computation Methods. In *Computat*, pages 111–116. Physica-Verlag HD, Heidelberg.
- Brandt, M. J., IJzerman, H., Dijksterhuis, A., Farach, F. J., Geller, J., Giner-Sorolla, R., Grange, J. a., Perugini, M., Spies, J. R., and van ’t Veer, A. (2014). The Replication Recipe: What makes for a convincing replication? *Journal of Experimental Social Psychology*, 50(1):217–224.
- Chib, S. and Greenberg, E. (1995). Understanding the Metropolis-Hastings Algorithm. *The American Statistician*, 49(4):327.
- Chib, S. and Jeliazkov, I. (2001). Marginal Likelihood From the Metropolis-Hastings Output. *Journal of the American Statistical Association*, 96(453):270–281.
- Cumming, G. (2012). *Understanding The New Statistics: Effect Sizes, Confidence Intervals, and Meta-Analysis*. Routledge, New York.
- Dienes, Z. (2011). Bayesian Versus Orthodox Statistics: Which Side Are You On? *Perspectives on Psychological Science*, 6(3):274–290.
- Dienes, Z. (2016). How Bayes factors change scientific practice. *Journal of Mathematical Psychology*, pages 1–29.
- Edwards, W., Lindman, H., and Savage, L. J. (1963). Bayesian statistical inference for psychological research. *Psychological Review*, 70(3):193–242.
- Gelman, A., Lee, D., and Guo, J. (2015). Stan: A Probabilistic Programming Language for Bayesian Inference and Optimization. *Journal of Educational and Behavioral Statistics*, 40(5):530–543.

- Goschke, T. and Dreisbach, G. (2008). Conflict-Triggered Goal Shielding: Response Conflicts Attenuate Background Monitoring for Prospective Memory Cues. *Psychological Science*, 19(1):25–32.
- Han, C. and Carlin, B. P. (2001). Markov Chain Monte Carlo Methods for Computing Bayes Factors. *Journal of the American Statistical Association*, 96(455):1122–1132.
- Hitchcock, D. B. (2003). A History of the Metropolis–Hastings Algorithm. *The American Statistician*, 57(4):254–257.
- Jeffreys, H. (1961). *Theory of Probability*. Clarendon Press, Oxford, 3. ed. edition.
- Kruschke, J. K. (2015). Introduction: Credibility, Models, and Parameters. In *Doing Bayesian Data Analysis: A Tutorial with R, JAGS, and Stan*, chapter 2, pages 15–32. Academic Press / Elsevier, Waltham, MA, 2nd ed. edition.
- Lakens, D. (2013). Calculating and reporting effect sizes to facilitate cumulative science: a practical primer for t-tests and ANOVAs. *Frontiers in Psychology*, 4(NOV):1–12.
- Lecoutre, B. (1999). Two useful distributions for Bayesian predictive procedures under normal models. *Journal of Statistical Planning and Inference*, 79(1):93–105.
- Lindley, D. V. (1993). The Analysis of Experimental Data: The Appreciation of Tea and Wine. *Teaching Statistics*, 15(1):22–25.
- McElreath, R. (2016). *Statistical Rethinking: A Bayesian Course with Examples in R and Stan*. CRC Press.
- Morey, R. D., Romeijn, J.-W., and Rouder, J. N. (2016). The philosophy of Bayes factors and the quantification of statistical evidence. *Journal of Mathematical Psychology*.
- Open Science Collaboration (2015). Estimating the reproducibility of psychological science. *Science*, 349(6251):aac4716–aac4716.
- Poitevineau, J. and Lecoutre, B. (2010). Implementing Bayesian predictive procedures: The K-prime and K-square distributions. *Computational Statistics and Data Analysis*, 54(3):724–731.
- Rouder, J. N. and Morey, R. D. (2012). Default Bayes Factors for Model Selection in Regression. *Multivariate Behavioral Research*, 47(6):877–903.
- Rouder, J. N., Morey, R. D., Speckman, P. L., and Province, J. M. (2012). Default Bayes factors for ANOVA designs. *Journal of Mathematical Psychology*, 56(5):356–374.
- Rouder, J. N., Speckman, P. L., Sun, D., Morey, R. D., and Iverson, G. (2009). Bayesian t tests for accepting and rejecting the null hypothesis. *Psychonomic Bulletin & Review*, 16(2):225–237.

- Schönbrodt, F. D., Wagenmakers, E.-J., Zehetleitner, M., and Perugini, M. (2015). Sequential Hypothesis Testing With Bayes Factors: Efficiently Testing Mean Differences. *Psychological Methods*.
- Simmons, J. P., Nelson, L. D., and Simonsohn, U. (2011). False-Positive Psychology: Undisclosed Flexibility in Data Collection and Analysis Allows Presenting Anything as Significant. *Psychological Science*, 22(11):1359–1366.
- Simonsohn, U. (2015). Small Telescopes: Detectability and the Evaluation of Replication Results. *Psychological Science*, 26(5):559–569.
- Steiger, J. H. (2004). Beyond the F Test: Effect Size Confidence Intervals and Tests of Close Fit in the Analysis of Variance and Contrast Analysis. *Psychological Methods*, 9(2):164–182.
- Verhagen, J. and Wagenmakers, E.-J. (2014). Bayesian tests to quantify the result of a replication attempt. *Journal of Experimental Psychology: General*, 143(4):1457–1475.
- Wasserstein, R. L. and Lazar, N. A. (2016). The ASA’s Statement on p -Values: Context, Process, and Purpose. *The American Statistician*, 70(2):129–133.
- Williams, L. E. and Bargh, J. a. (2008). Keeping One’s Distance: The Influence of Spatial Distance Cues on Affect and Evaluation. *Psychological Science*, 19(3):302–308.